MENG 293 Final Project Technical Document

BMW E28 Chassis Tow Hitch Receiver

Brian Gill
C0503517

Table of Contents

Introduction

Purpose

This project is meant to promote renewable behaviours through repair and improvement over discard and replacement. To meet this goal, this document describes the development of a solution to enable the BMW e28 chassis to tow trailered loads with a standard ball hitch.

Problem Definition

Original e28 chassis do not come equipped with tow hitch receivers. BMW has previously offered a solution for their European vehicles with a tow mass rating of 1800kg [7]; however, this product is long since discontinued, and was quite intrusive into the trunk area with large bolts and plates to clamp down on the sheet metal, supports stretching from under the taillamps to the tops of the rear strut towers, and a long, complicated install procedure. There are no current aftermarket solutions for these cars. The problem to solve is offering a simple tow hitch receiver that does not modify the chassis and matches the original capacity.

Summary

This document describes the development of a trailer hitch receiver for the BMW e28 chassis. In the design process analysis was done to determine a minimum strength of the chassis where the receiver is to mount. To determine the minimum strength, calculations were done based on bumper safety standards applicable to this vehicle. Comparing the maximum stresses calculated from towing at rated capacity versus the minimum stresses based on required impact ratings, it was found that the chassis cannot be damaged through towing at maximum capacity. The maximum stresses calculated are based on a handful of assumptions and neglected factors which aim to exaggerate the calculated values to add a layer of safety for the design. Although the chassis won't be damaged by an 1800kg unbraked load it is not recommended as the maximum deceleration is significantly decreased and could be dangerous in the event of an emergency stop.

Methodology

Measured Values

Several values needed to be measured for both calculations and design decisions. For the receiver to mate to the existing chassis and fit between the exhaust and bottom of the bumper, the bumper shock absorber flange profile, spacing, and compression distance were measured. To estimate the frictional coefficient of tires, acceleration values at maximum braking were measured. A 3D scan of the chassis was taken to test fit the model in software prior to any assembly.

Calculated Values

The maximum normal acceleration withstood by tires was calculated by recording the velocity where grip would turn to slip through a corner and measuring the radius of the corner at that time. The average deceleration in impact tests was calculated using the displacement of the bumper shock absorbers, and the speed of the impact test. The centre of gravity of the e28 chassis was calculated using the weight distribution, and wheelbase of the car. Equations were developed to determine the maximum straight-line loads on the hitch receiver based on tow mass, road grade, and tire friction limits.

Assumptions

To solve some of the equations developed, assumptions had to be made. The intent of the assumptions made is to increase the estimated forces on the receiver, decrease the chassis rated minimum stresses, or to simplify calculations due to variables that frequently change. The following assumptions were made and held consistent throughout the calculations.

Trailered load is 1800kg mass.
Tongue carries 10% of trailered weight.
Trailer axle to tongue length is 2.45m.
Trailered load is slender and has negligible aerodynamic drag even at high speeds.
Trailered center of mass, tow ball, and vehicle centre of mass are all 0.6m above ground level.
Tires have a static frictional coefficient of 0.80.
Trailer axle has a static rolling resistance coefficient of 0.015 [1].
Unpowered, non-braking chassis axles have a static rolling resistance of 0.015 [1].
Deceleration during a crash tests is constant.

Neglected Factors

The purpose of neglecting the following factors is to simplify the calculations and bias them towards higher stresses throughout the system. Neglecting the slight increase of tire friction from wheel slip implies that when towing a load, the driver will act responsibly without slipping any tires.

Aerodynamic drag of the vehicle.
Vehicle weight transfer from brake dive and acceleration squat.
Vehicle weight transfer from transverse loading.
Increased tire friction between 5%- and 20%-wheel slip.
Brake bias and force limits of the braking system.
Acceleration limits from the engine.

Establishing a Baseline

Chassis and Receiver Strength Minimums

Analysis of the chassis has been done based on the test procedures and requirements for bumpers outlined in [5]. Based on Table 1 and the fact that the North American e28 chassis remained unchanged through all its production starting model year 1981, the minimum bumper impact ratings can be summarized as the chassis remains undamaged following a 5 MPH straight forward front and rear impact or a 3 MPH impact at an angle of 60° from the centre line of the car. The claim to an undamaged chassis follows from a list of definitions written in [5].

The patent describing the rear bumper shock mechanism [2] describes different dimensions from what is measured on the vehicle; however the shape of graph 2 depicting load vs stroke tells us that there is a roughly linear relation between stroke distance and reaction force, which by extension would be seen as a linear increase of deceleration during a crash.

The original tow hitch receiver is commonly rated at 1800kg online [7], although a reputable source has yet to be found. For the European version of this vehicle, a sales brochure translated from Dutch using Duck Duck Go's translator claims "Permissible trailer weight braked 1400 kg up to max. 12% incline, unbraked 500 kg (trailer weight increase possible, please contact your BMW dealer)." [6]. The mention of possible weight increase supports the claim of a maximum of 1800kg. For the calculations done, the assumption follows an 1800kg load without brakes.

Comparison of Calculated and Measured Values

To ensure the values used as the baseline in the calculations are reasonable, this is a comparison of calculated and measured values using data from multiple sources. In the following text g's are used in reference to acceleration, 1 g is $9.81 m/s^2$, and coefficient before g can always be translated as the coefficient of friction between the tire and road surface. Maximum deceleration has been measured at 0.80 g's in a test vehicle; the performance model e28 m5 manages an average of 0.97 g's, (1) using data from info sheet 1. Readings from [1] summarize a technical document outlining requirements for braking system performance, of which the greatest deceleration value is 20 ft/s^2 (0.62 g's). Historical data combined with a measurement from Screenshot 1 provides a lateral acceleration of 0.80 g's (6). The test vehicle used measured lateral acceleration at 0.60 g's. Given that [3] is using performance tires and an improved braking system, the slightly higher deceleration is logical.

Chassis Maneuverability Limitations

Every vehicle's maneuverability is ultimately limited by the tires. Based on 0.80 g's of acceleration for both straight line and lateral motion, the coefficient of static friction between the tires and the road surface can be determined as 0.80. That is to say that in any position the net acceleration of the vehicle can not exceed 0.80 g's. This limit decreases once the trailered load is considered, although the coefficient of friction for the driving vehicles tires does not change. The calculations done to determine the maximum loads present during trailering are in the scenario of perfect braking. Events of large forward and lateral acceleration would be considered irresponsible during towing and as such there are no calculations in this report covering those scenarios.

Design

Description

The primary goal of the design is to enable users to safely tow any cargo up to 1800kg mass. Beyond reaching this target the design focuses on fixing some issues prevalent in the OEM offered solution from many years ago, these improvements are directed at not impacting the functionality of any safety equipment, not being an eyesore, and not protruding into the trunk area or modifying the chassis in any other way. Featured in the design is safety hoops for safety chains to be installed on, a wire run routing that forces a drip loop into the harness, and a large split to even out the reaction forces and stresses developed during towing. The design is on it's third revision after the first major change. Future changes are limited to adjusting the split to further improve the load balancing and the designated failure point.

Materials

The goals of specifying materials for this design are primarily to reduce cost, and secondly to prevent corrosion. For all make parts designed the material is mild steel, this is because mild steel is easy to weld, cheap, and strong. All installation hardware should be stainless steel to prevent corrosion. The receiver should be powder coated to withstand the harsh conditions present during road travel.

Safety

For this receiver to have no interference with the original safety equipment, when installed without a tow ball the receiver must not protrude past the distance of an uncompressed bumper. There must be no way for a trailered load of 1800kg or less to damage the chassis. The electrical system to provide power for running, brake, and turn signal lamps must be fully sealed where it leaves the chassis, be properly sleeved to prevent abrasion, and have drip loops to ensure water does not collect in the connectors.

Model Images and Drawings

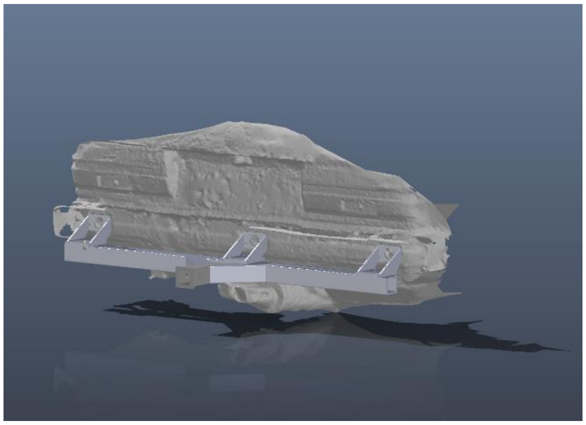


figure 1: Model version 1A [created by author]

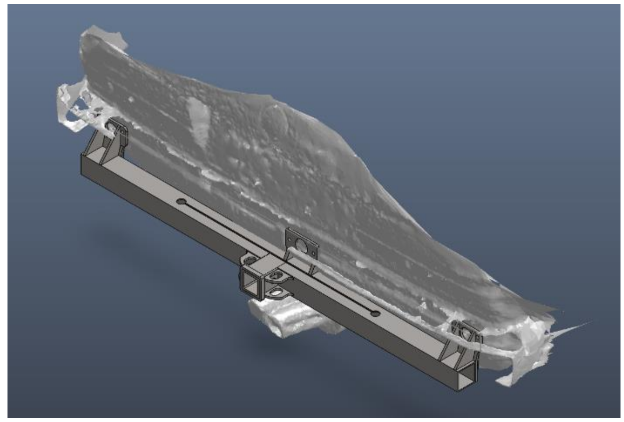


figure 2: Model version 2B [created by author]

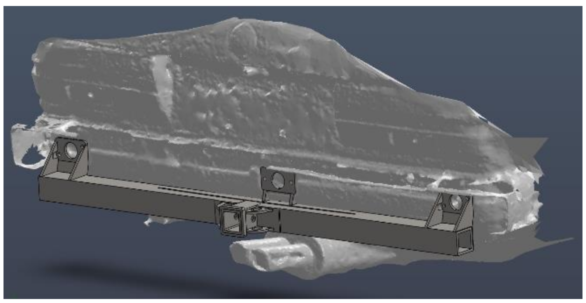


figure 3: Model version 2C [created by author]

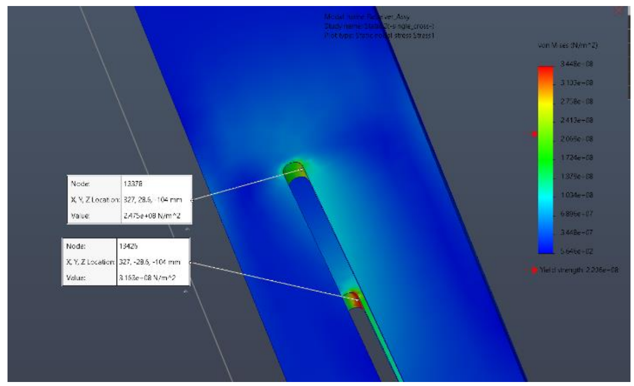


figure 4: Point of failure [created by author]

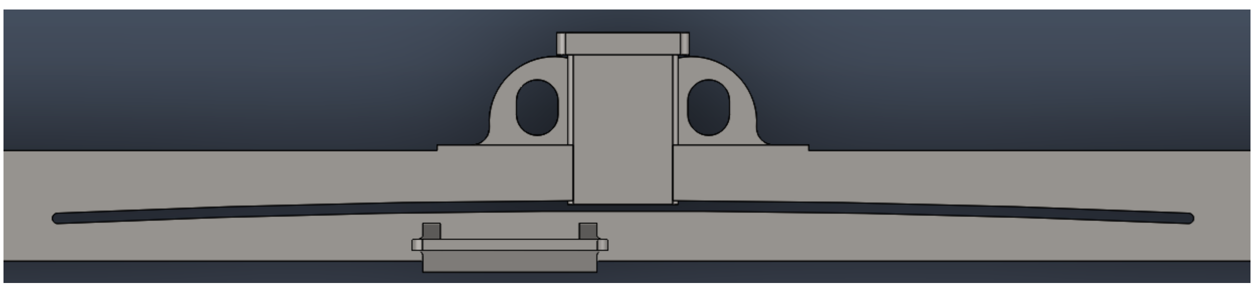


figure 5: Relief for stress management in version 2C [created by author]

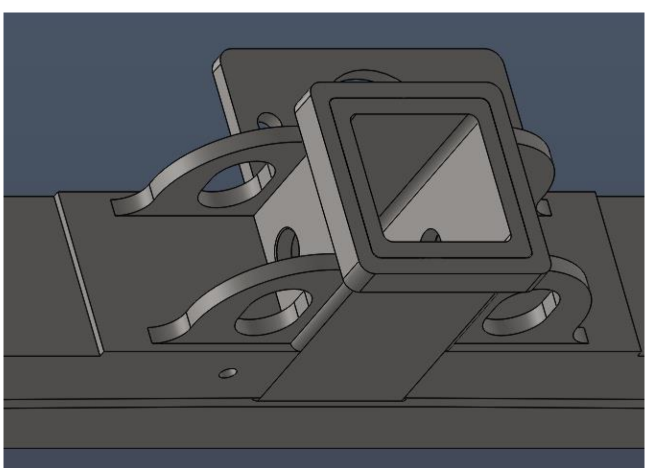


figure 6: wire exit on bottom forces drip loop [created by author]

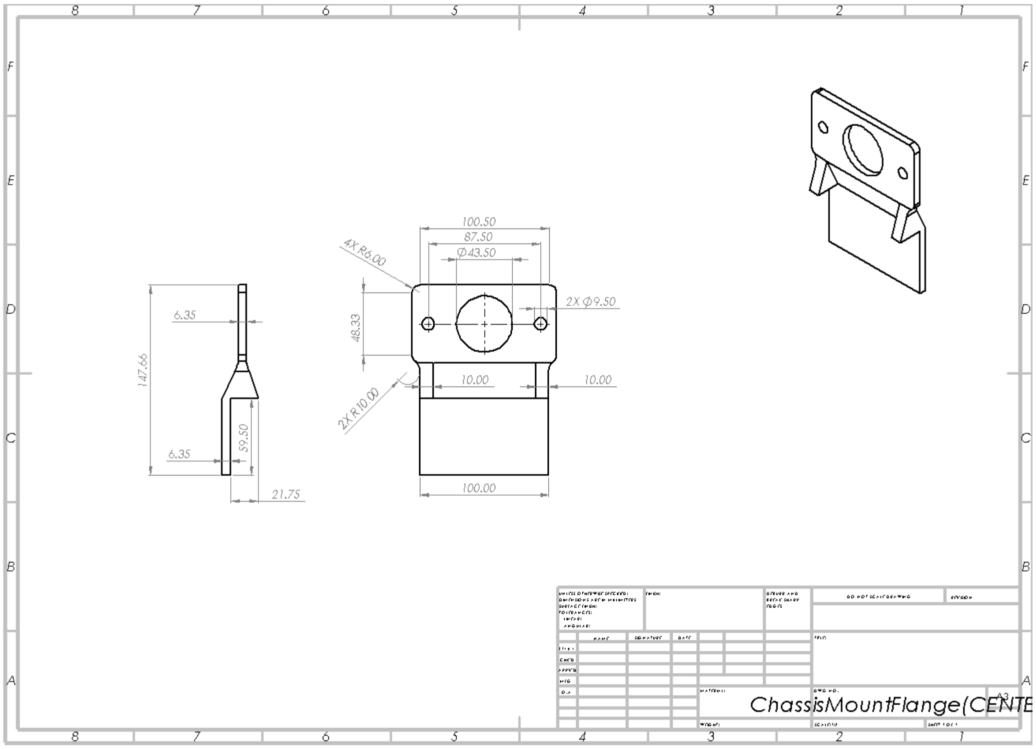


figure 7: Center chassis mount drawing [created by author]

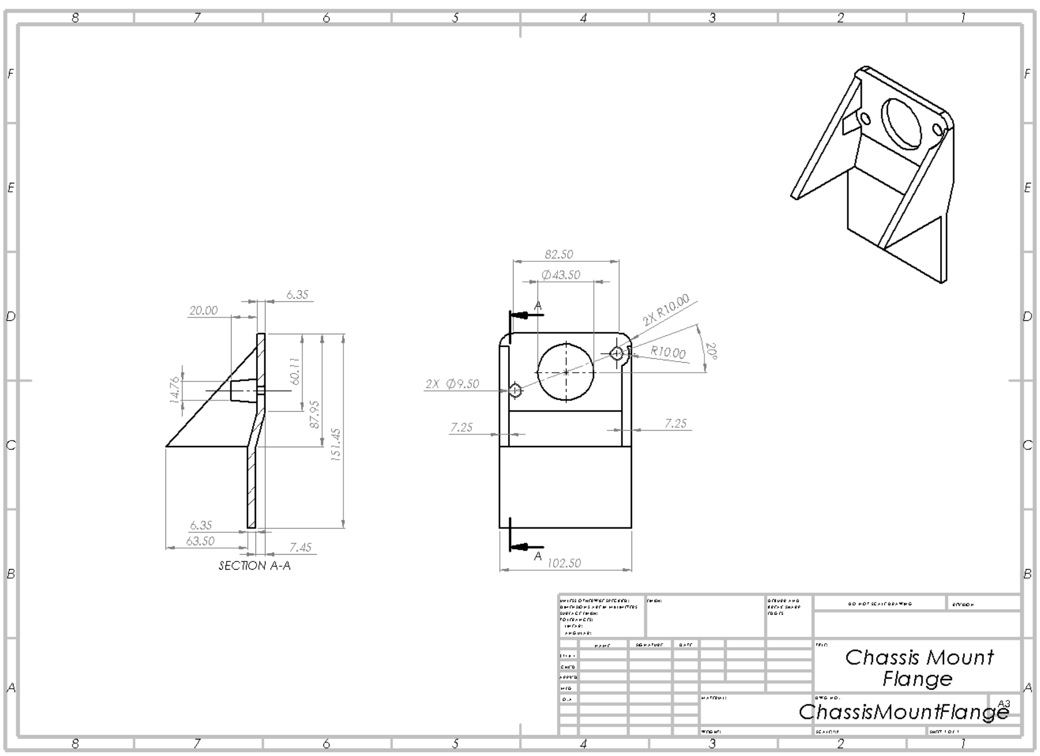


figure 8: Outside chassis mount drawing [created by author]

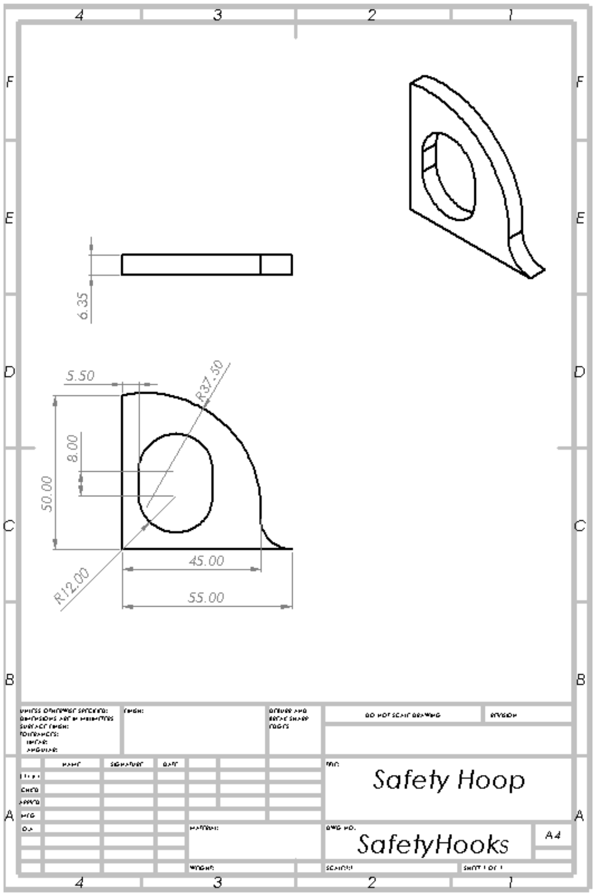


figure 9: Safety hoop drawing [created by author]

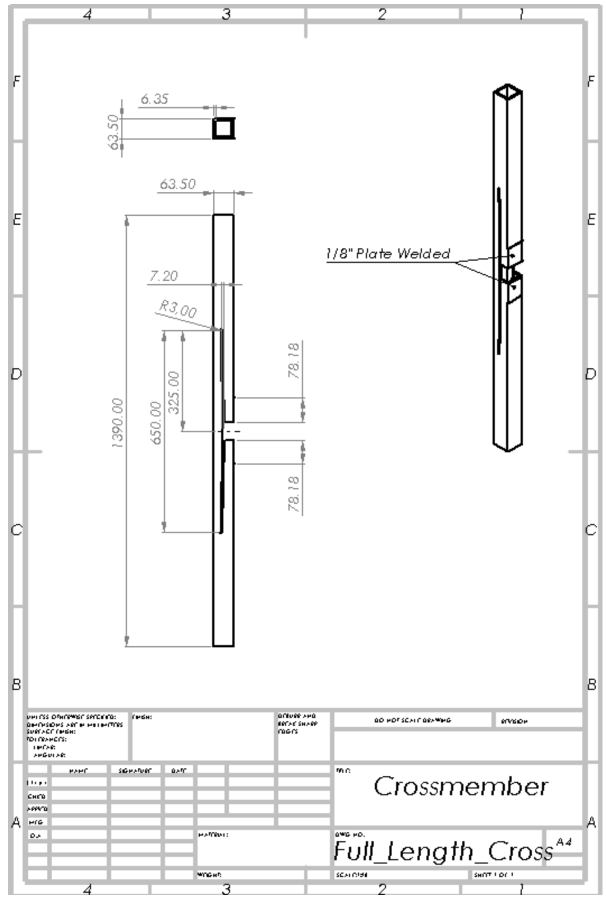


figure 10: Crossmember drawing [created by author]

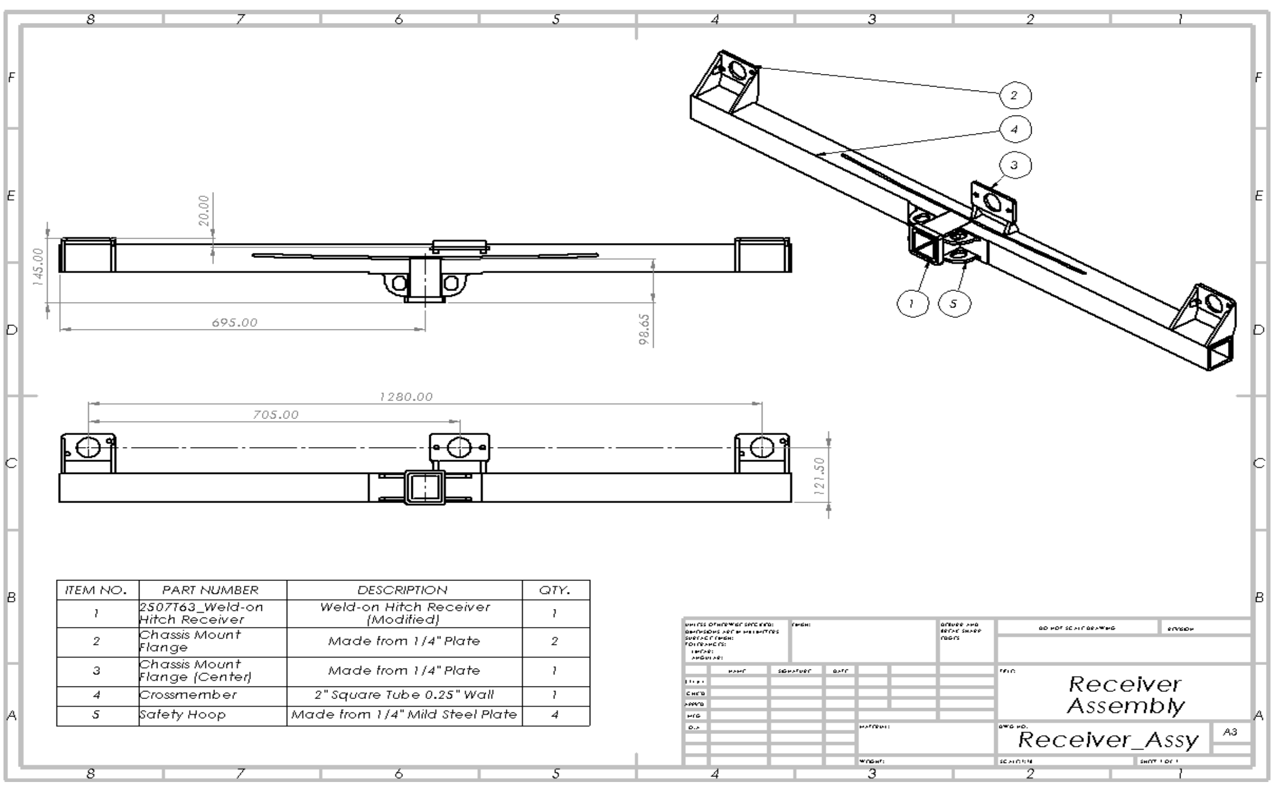


figure 11: Receiver assembly drawing [created by author]

Calculations

Deceleration of performance model E28 M5:

Maximum Braking Tests from External Source [3]: $100KPH = 27.778\,m/s\,, \Delta t = 2.9s$

$$a = \frac{\Delta v}{\Delta t} = \frac{27.778\frac{m}{s} - 0\frac{m}{s}}{2.9s} = 9.579\,\frac{m}{s^2} \quad (1)$$

Deceleration and forces from bumper impact requirements:

5 MPH impact: $5MPH = 2.2352\,m/s\,, \Delta s = 0.075m$

$$a = \frac{V_B^2 - V_A^2}{2 \times \Delta s} = \frac{0 - (2.2352\frac{m}{s})^2}{2 \times 0.075m} = -33.3075\,\frac{m}{s^2} \quad (2)$$

$$F_{net} = m \times a = 1400kg \times 33.3075\,\frac{m}{s^2} = 46630N \quad (3)$$

3 MPH corner impact: $3MPH = 1.3411\,m/s\,, \Delta s = 0.075m$

$$a = \frac{V_B^2 - V_A^2}{2 \times \Delta s} = \frac{0 - (1.3411\frac{m}{s})^2}{2 \times 0.075m} = -11.9903\,\frac{m}{s^2} \quad (4)$$

$$F_{net} = m \times a = 1400kg \times 11.9903\,\frac{m}{s^2} = 16786N \quad (5)$$

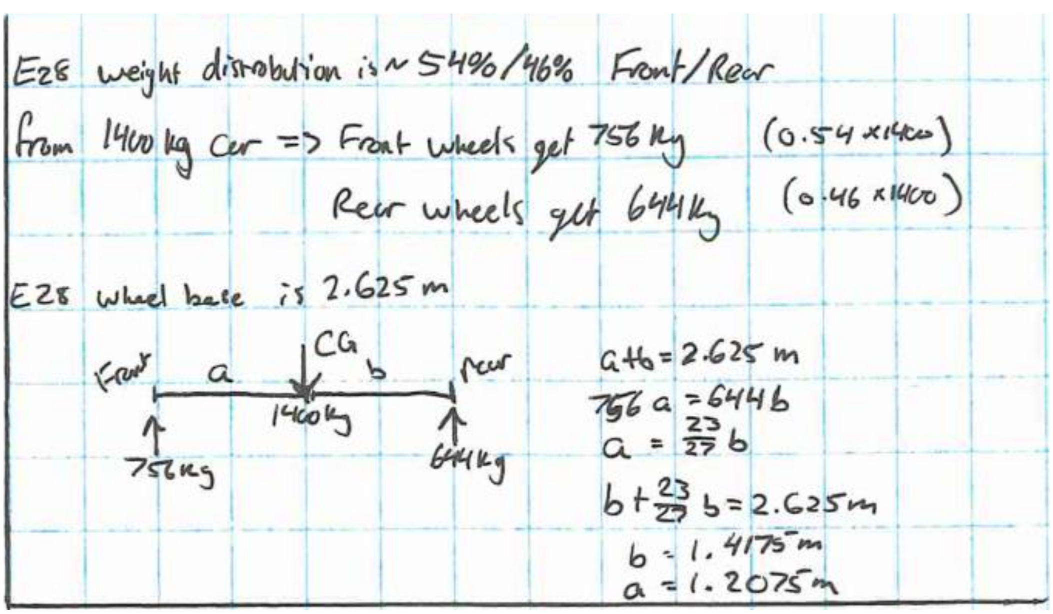
$$(\,inline\ with\ the\ car, \div \cos(30)\ to\ get\ total\ force = 19{,}384N\,)$$

Maximum lateral acceleration:

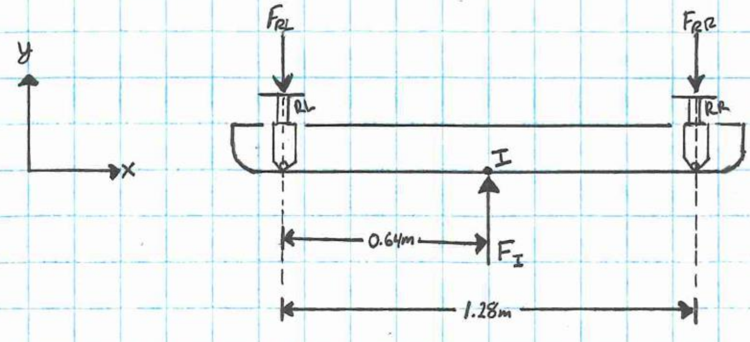
Curvature radius: 50m [8], maximum velocity: 70km/h (19.44m/s)

$$a_n = \frac{v^2}{r} = \frac{(19.44\frac{m}{s})^2}{50m} = 7.5617\,\frac{m}{s^2} \quad (6)$$

Center of gravity of the chassis:

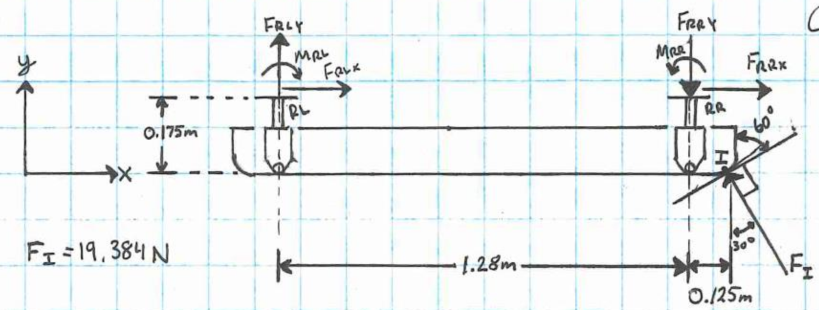


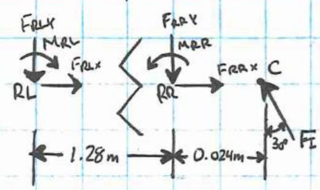
Bumper impact reaction forces:



Direct impact from 5 MPH
Evenly distributed across the bumper equates to F_I at the center.
Symmetry about I provides
$F_{RL} = F_{RR}$, $F_I = 46{,}630$ N
$\Sigma F_y = 0$ provides $F_{RL} = F_{RR} = 23{,}315$ N

Corner impact from 3 MPH





$F_I = 19{,}384$ N

F_I intersection with F_{RRX}, F_{RLX} axis.
F_{RLY} arrow is backwards.

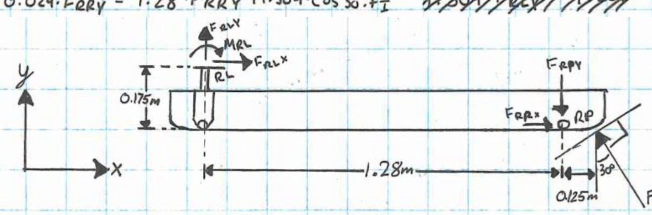
~~$\Sigma M_{RR} = 0$ $0.125m \times F_I \times \cos 30 - 0.175 \times F_I \times \sin 30 - 1.28 \times F_{RRY} = 0$~~

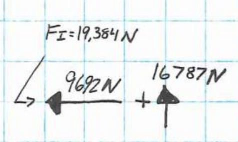
$\Sigma F_y = 0$ $F_{RLY} + F_I \cos 30 - F_{RRY} = 0$ $F_{RRY} - F_{RLY} = \cos 30 \times F_I$

$\Sigma M_C = 0$ $M_{RR} - M_{RL} + 0.024m \times F_{RRY} - 1.304m \times F_{RLY} = 0$ $M_{RR} - M_{RL} = 1.304 \cdot F_{RLY} - 0.024 \cdot F_{RRY}$

$\Sigma M_{RL} = 0$ $M_{RR} - M_{RL} - 1.28m \times F_{RRY} + 1.304m \times \cos 30 \times F_I = 0$ $M_{RR} - M_{RL} = 1.28 F_{RRY} + 1.304 \cdot \cos 30 \cdot F_I$

$1.304 \cdot F_{RLY} - 0.024 \cdot F_{RRY} = 1.28 \cdot F_{RRY} + 1.304 \cdot \cos 30 \cdot F_I$





$\Sigma M_{RP} = 0$ $16787N \times 0.125m - M_{RL} - 1.28m \times F_{RLY} - 0.175m \times F_{RLX} = 0$

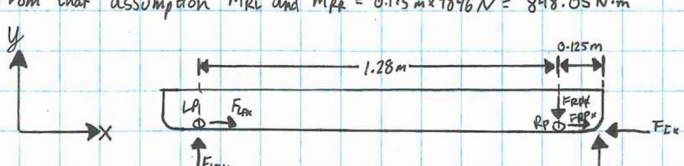
$\Sigma F_Y = 0$ $F_{IY} + F_{RLY} - F_{RPY} = 0$ $F_{RLY} = F_{RPY} - F_{IY} = 18426.36N - 16787N$ $F_{RLY} = 1639.36N$ (Arrow points wrong direction)

$2098.375\ Nm - M_{RL} - 1.28m \cdot 1639.36N - 0.175m \times F_{RLX} = 0 \Rightarrow M_{RL} - 0.175m \cdot F_{RLX} = 0$ $M_{RL} = 0.175m \times F_{RLX}$

The same work can be done to show that $M_{RR} = 0.175m \times F_{RRX}$ and $F_{RRY} = F_{RPY} = 18426.36N$

F_{RLX} and F_{RRX} unsolvable. To minimize stresses must assume $F_{RLX} = F_{RRX} = \frac{9692N}{2} = 4846N$

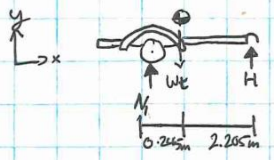
From that assumption M_{RL} and $M_{RR} = 0.175m \times 4846N = 848.05\ N \cdot m$

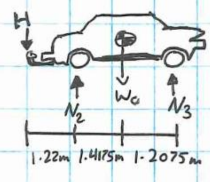


$\Sigma M_{RP} = 0$ $0.125m \times F_{IY} - 1.28m \times F_{LPY} = 0$ $F_{LPY} = \frac{0.125m \times F_{IY}}{1.28m}$ $F_{IY} = 16787\ N$ $F_{LPY} = \frac{0.125m \cdot 16787N}{1.28m} = 1639.36N$

$\Sigma F_Y = 0$ $F_{IY} + F_{LPY} - F_{RPY} = 0$ $F_{RPY} = F_{IY} + F_{LPY} = 16787N + 1639.36\ N$ $F_{RPY} = 18426.36N$

Equations governing hitch forces:





$H = 0.1 \cdot W_t = 0.1 \times 1800 \times 9.81$

$H = 1765.8N$

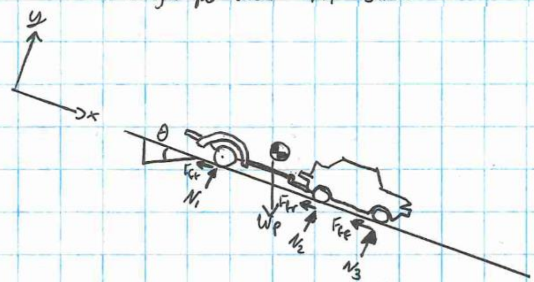
$W_c = 1400\,kg \times 9.81\,\frac{m}{s^2} = 13,734N$

$\Sigma M_{N_2} = 0 \quad 1.22 \times H + 2.625 \times N_3 = 1.4175 \times W_c \qquad N_3 = \frac{1.4175 \cdot W_c - 1.22 \cdot H}{2.625} = 6595.683429N$

$\Sigma F_y = 0 \quad N_2 + N_3 = W_c + H \quad N_2 = W_c + H - N_3 = 8904.116571N$

Total weight $= W_c + W_t = 31,392$ N

% of weight per wheel: $N_1 = \frac{0.4 \cdot W_t}{31,392} = 50.625\%$ $\quad N_2 = \frac{8904}{31,392} = 28.364\%$ $\quad N_3 = \frac{6595}{31,392} = 21.011\%$



Based on the above analysis

$N_1 = 31,392 \times 0.50625 \times \cos(\theta)$

$N_2 = 31,392 \times 0.28364 \times \cos(\theta)$

$N_3 = 31,392 \times 0.21011 \times \cos(\theta)$

As well as an additional force in the X-direction equal to $31,392 \times \sin(\theta)$

$F_{ft} = 0.15 \times N_1 = 0.15 \times 0.50625 \times 31392 \times \cos(\theta)$ (0.015)

$F_{fr} = 0.8 \times N_2 = 0.80 \times 0.28364 \times 31392 \times \cos(\theta)$

$F_{ff} = 0.8 \times N_3 = 0.80 \times 0.21011 \times 31392 \times \cos(\theta)$

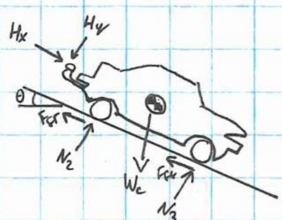
$\Sigma F_x = m \cdot a_x \quad 31,392 \times \sin(\theta) - 31392 \times 0.15 \times 0.50625 \times \cos(\theta) \ldots$

$\to 31,392 \times [\sin(\theta) - \cos(\theta)(0.15 \times 0.50625 + 0.80 \times (0.28364 + 0.21011))]$ (0.015)

$a_x = [\sin(\theta) - \cos(\theta) \times (0.40259375)] \times 9.81$

9% Grade $\Rightarrow a_x = -3.0542\,\frac{m}{s^2}$

Flat Road $\Rightarrow a_x = -3.949\,\frac{m}{s^2}$



$m \cdot a_x = \Sigma F_x$

$1400 \times 9.81 \times [\sin(\theta) - \cos(\theta) \times 0.40259375] = H_x + W_c \times \sin(\theta) - F_{ff} - F_{fr}$ (13,734)

$-H_x = W_c \cdot \sin(\theta) - \text{"} - F_{ff} - F_{fr} \qquad W_c = 9.81 \times 1400 = 13,734$

$-H_x = 13,734 \times \sin(\theta) - \text{" "} - 12,349.84 \times \cos(\theta)$

$-H_x = 13,734[\sin(\theta) - \sin(\theta) + \cos(\theta) \times 0.40259375] - 12,349.84 \times \cos(\theta)$

$-H_x = 5529.2225663 \times \cos(\theta) - 12,349.84 \times \cos\theta$

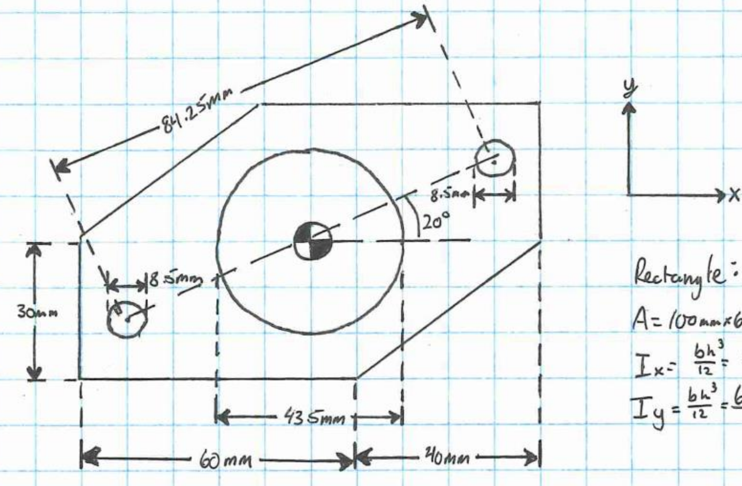
$-H_x = -6870.617438 \times \cos(\theta)$

9% Grade $\Rightarrow H_x = 6842.96$ N

Flat Road $\Rightarrow H_x = 6870.62$ N (Maximum)

Note: at 1800kg the maximum deceleration is 3.949m/s^2 (0.40 g's) which is below the minimum deceleration values for road vehicles presented in [1]; because of this towing unbraked loads at or near the rated capacity of 1800kg is not recommended.

Geometry analysis of the bumper shock flange and stress calculations:



Rectangle:

$A = 100mm \times 60mm = 6000 mm^2$

$I_x = \frac{bh^3}{12} = \frac{100 \times 60^3}{12} = 1,800,000 \; mm^4$

$I_y = \frac{bh^3}{12} = \frac{60 \times 100^3}{12} = 5,000,000 \; mm^4$

Big Circle:

$A = \pi \left(\frac{43.5}{2}\right)^2 = 1486.1697 mm^2$

$I_x = I_y = \frac{\pi}{4} r^4 = \left(\frac{\pi}{4}\right)\left(\frac{43.5}{2}\right)^4 = 175,762.7854 mm^4$

$dx = \left(\frac{84.25}{2}\right)\cos 20 = 39.5846 mm \quad dy = \left(\frac{84.25}{2}\right)\sin 20 = 14.4076 mm$

Small circle:

$A = \pi\left(\frac{8.5}{2}\right)^2 = 56.7450 mm^2$

$I_x = I_y = \left(\frac{\pi}{4}\right)\left(\frac{8.5}{2}\right)^4 = 256.2392 mm^4$

Triangle:

$A = \frac{b \cdot h}{2} = \frac{40 \cdot 30}{2} = 600 mm^2 \quad dx = 10 + \left(\frac{2}{3}\right)40 = 36.\overline{66} mm \quad dy = \left(\frac{2}{3}\right)30 = 20 mm$

$I_x = \frac{b \cdot h^3}{36} = \frac{40 \cdot 30^3}{36} = 30,000 mm^4 \quad I_y = \frac{h \cdot b^3}{36} = \frac{30 \cdot 40^3}{36} = 53,333.\overline{33} mm^4$

$I_x = I_{rect} - I_{big\,O} - 2(I_{small\,O} + A_{small\,O} \cdot dy^2) - 2(I_{tri} + A_{tri} \cdot dy^2)$

$= 1,800,000 - 175,762.7854 - 2(256.2392 + 56.745 \cdot 14.4076^2) - 2(600 \cdot 20^2 + 30,000)$

$I_x = 1,060,166.603 \; mm^4$

$I_y = I_{rect} - I_{big\,O} - 2(I_{small\,O} + A_{small\,O} \cdot dx^2) - 2(I_{tri} + A_{tri} \cdot dy^2)$

$= 5,000,000 - 175,762.7854 - 2(256.2392 + 56.745 \cdot 39.5846^2) - 2(600 \cdot 36.\overline{66}^2 + 53,333.\overline{33})$

$I_y = 2,926,215.963 \; mm^4$

$Area_{total} = Rect - big\,O - 2(small\,O) - 2(tri) = 6000 - 1486.197 - 2(56.7450) - 2(600)$

$A = 3200.313 \; mm^2$

$\sigma_{avg} = \frac{N}{A} \quad \sigma_{(y)} = \frac{M \cdot y}{I}$

5MPH straight: 23,315 N Normal (Compression), No moment

3MPH Corner: 18426.36 N (C) + 848.05 N·m, 1639.36 (T) + 848.05 N·m

$y_{max} = 30 mm \quad x_{max} = 50 mm$

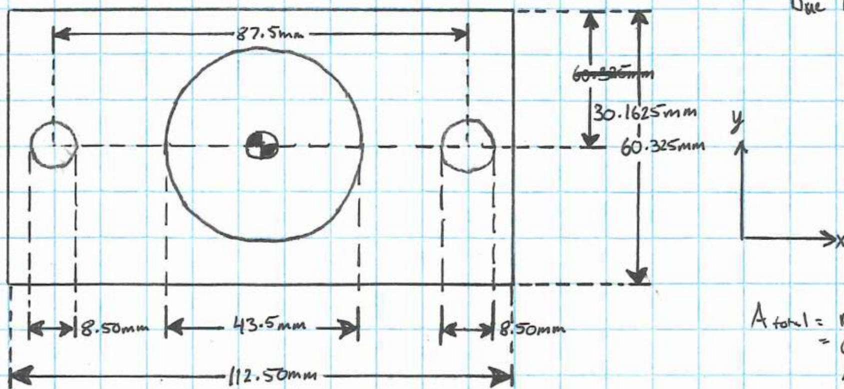
5MPH $\sigma = \frac{-23,315 N}{3200.313 mm^2} = 7.2852 MPa$ Compression

3MPH $\sigma_1 = \frac{-18426.36}{3200.313} \pm \frac{848.05 \cdot 50 \cdot 1000}{2,926,215.963} =$ ~~[illegible]~~) Compression 20.2482 MPa (max)

$\sigma_2 = \frac{+1639.36}{3200.313} \pm \frac{848.05 \cdot 50 \cdot 1000}{2926215.963} =$ ~~[illegible]~~) Tension 15.0029 MPa (max)

Geometry analysis of the hitch receiver mounting flanges:

Due to symmetry centroid remains in center



A_{total} = Rect − Big circle − 2(small circle)
= 6786.5625 − 1486.170 − 2(56.745)
A = 5186.9025 mm²

Area of big rectangle : $A = b \times h = 112.5 \times 60.325 = 6786.5625\,mm^2$ $I_x = \frac{b \cdot h^3}{12} = \frac{112.50 \times 60.325^3}{12} = 2{,}058{,}084.814\,mm^4$

Area of big circle : $A = \pi \frac{d^2}{4} = \pi\left(\frac{43.5^2}{4}\right) = 1486.170\,mm^2$ $I_x = \frac{\pi}{4} \cdot r^4 = \frac{\pi}{4}\left(\frac{43.5}{2}\right)^4 = 175{,}762.7854\,mm^4$

Area of small circle : $A = \pi \frac{d^2}{4} = \pi\left(\frac{8.5^2}{4}\right) = 56.745\,mm^2$ $I_x = \frac{\pi}{4} \cdot r^4 = \frac{\pi}{4}\left(\frac{8.5}{2}\right)^4 = 256.2392\,mm^4$

Everything lies on the neutral axis

$I_x = I_{rect} - I_{big\,circle} - 2(I_{small\,circle}) = 2058084.814 - 175762.7854 - 2(256.2392) = 1{,}881{,}809.55\,mm^4$

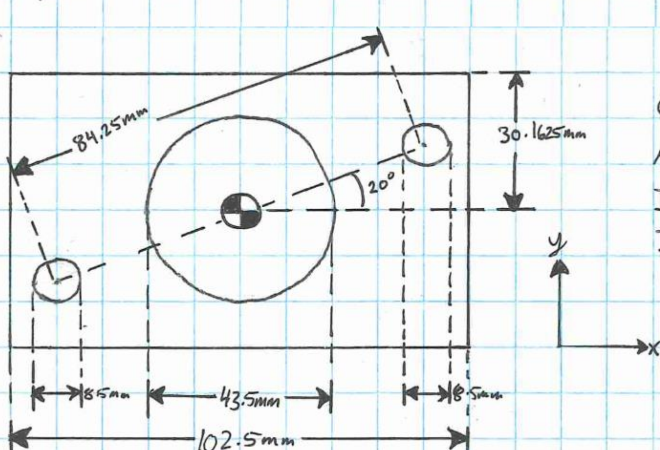
$I_{\square y} = 60.325 \cdot \frac{112.5^3}{12} = 7{,}157{,}702.637$ $I_{\circ y} = I_{\circ x}$

Small circles do not lie on neutral axis for I_y, still symetrical

$I_y = I_{\square} - I_{Big\circ} - 2(I_{small\circ} + A_{small\circ} \cdot dx^2)$ $dx = \frac{87.5}{2} = 43.75\,mm$

$I_y = 7{,}157{,}702.637 - 175{,}762.7854 - 2(256.2392 + 56.745 \times 43.75^2) =$ ~~6,974,987.383 mm⁴~~

$6{,}764{,}200.42\,mm^4$



Circle Areas and I_x/I_ys taken from before

Area of big rectangle: $A = b \cdot h = 102.5 \times 60.325 = 6183.3125$

$I_{\square x} = \frac{102.5 \times 60.325^3}{12} = 1{,}875{,}143.942\,mm^4$

$I_{\square y} = \frac{60.325 \times 102.5^3}{12} = 5{,}413{,}618.913\,mm^4$

Small circle $dx = \frac{84.25}{2} \times \cos(20) = 39.5846\,mm$

Small circle $dy = \frac{84.25}{2} \times \sin(20) = 14.4076\,mm$

$I_x = I_{\square} - I_{big\circ} - 2(I_{small\circ} + A_{small\circ} \cdot dy^2) = 1{,}875{,}143.942 - 175{,}762.7854 - 2(256.2392 + 56.745 \cdot 14.4076^2)$

$I_x = 1{,}675{,}310.545\,mm^4$

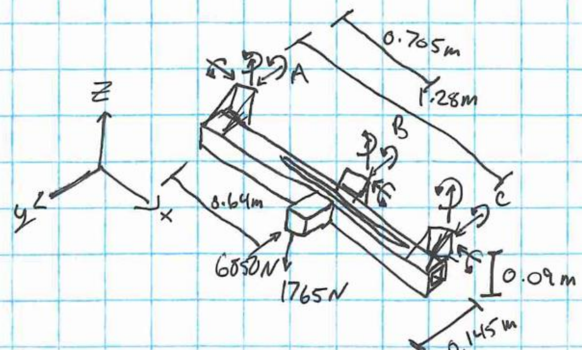
$I_y = I_{\square} - I_{big\circ} - 2(I_{small\circ} + A_{small\circ} \cdot dx^2) = 5{,}413{,}618.913 - 175{,}762.7854 - 2(256.2392 + 56.745 \cdot 39.5846^2)$

$I_y = 5{,}059{,}511.565\,mm^4$

A_{total} = Rect − Big ○ − 2(Small ○) = 6183.3125 − 1486.170 − 2(56.745)

A = 4583.6525 mm²

Stresses due to fully loaded receiver reactions:

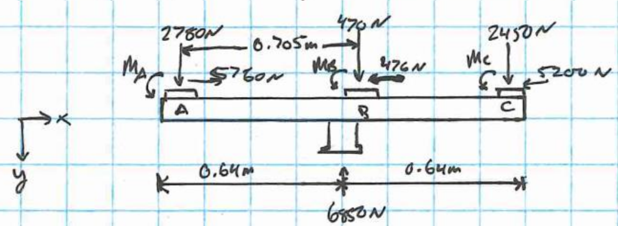


$A_y = 2780N$ $B_y =$ ~~1990N~~ $1470N$ $C_y = 2450N$
$A_x = 5670N$ $B_x = 476N$ $C_x = 5200N$
$A_z = 71.5N$ $B_z = 1990N$ $C_z = -293N$
} From Solidworks Static Study

Maximum moment around $x = 1765N \times 0.145m = 256\ N \cdot m$

Maximum moment around $Z = 6850N \times 0.64m = 4384\ N \cdot m$ (entirely impossible, should be close to 0)

Maximum moment around $y = 0$



$\circlearrowleft \sum M_A = 0$ $\quad 0.64m \times 6850N - 0.705m \times 470N - 1.28m \times 2450N + M_A + M_B + M_C = 0$

$M_A + M_B + M_C = 916.65\ N \cdot m$

↳ M_A and M_C should be close to eachother in magnitude, M_B should be close to 0.

Neglecting that and using $916.65\ N \cdot m$ to calculate stresses.

A stresses > C stresses.

A: $\sigma_{avg} \pm (\sigma_{(ymax)} + \sigma_{(xmax)})$

$$= \frac{-2780N}{4583.6525mm^2} \pm \left(\frac{916.65N \cdot m \cdot 1000 \cdot \frac{102.5mm}{2}}{5,059,511.565mm^4}\right) \pm \left(\frac{256Nm \cdot 1000 \cdot 30.1625mm}{1,675,310.545mm^4}\right) \Rightarrow$$

$\sigma_1 = 13.61$ MPa [Tension]

$\sigma_2 = 14.79$ MPa [Compression]

B: $\sigma_{avg} \pm (\sigma_{(ymax)} + \sigma_{(xmax)})$

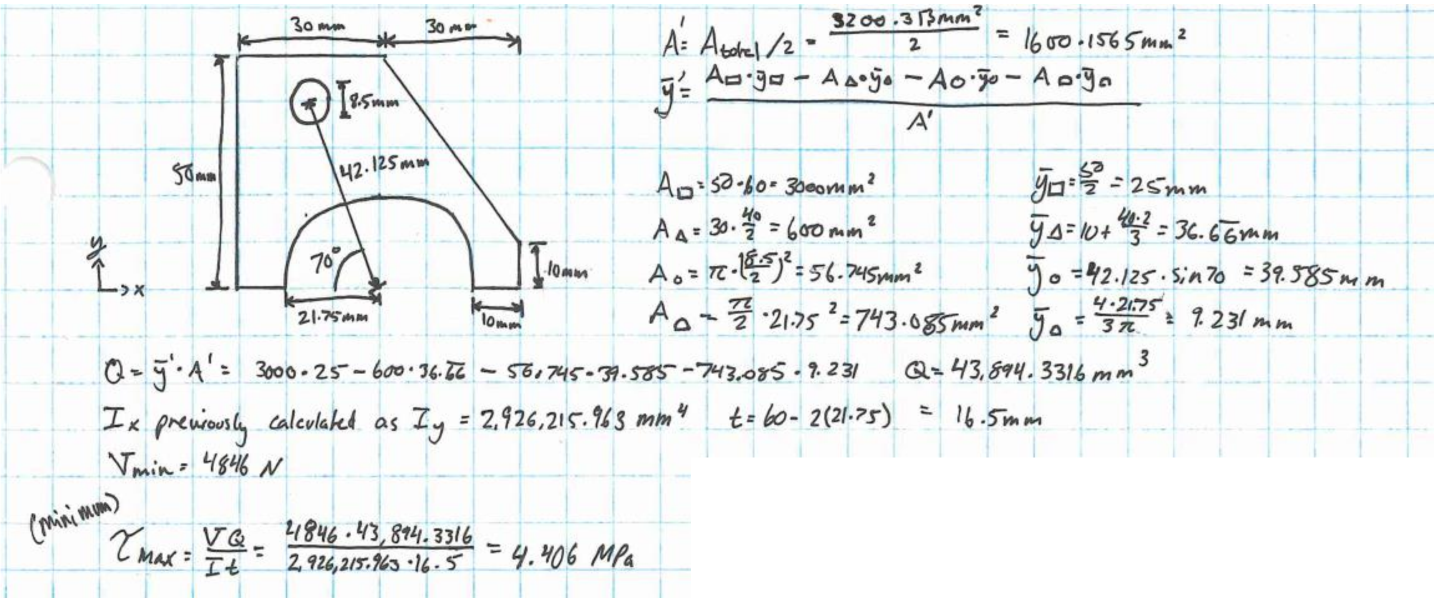
$$= \frac{-470N}{5186.9025mm^2} \pm \left(\frac{916.65N \cdot m \cdot 1000 \cdot \frac{112.5mm}{2}}{6,764,200.42mm^4} + \frac{256Nm \cdot 1000 \cdot 30.1625mm}{1,881,809.35mm^4}\right) \Rightarrow$$

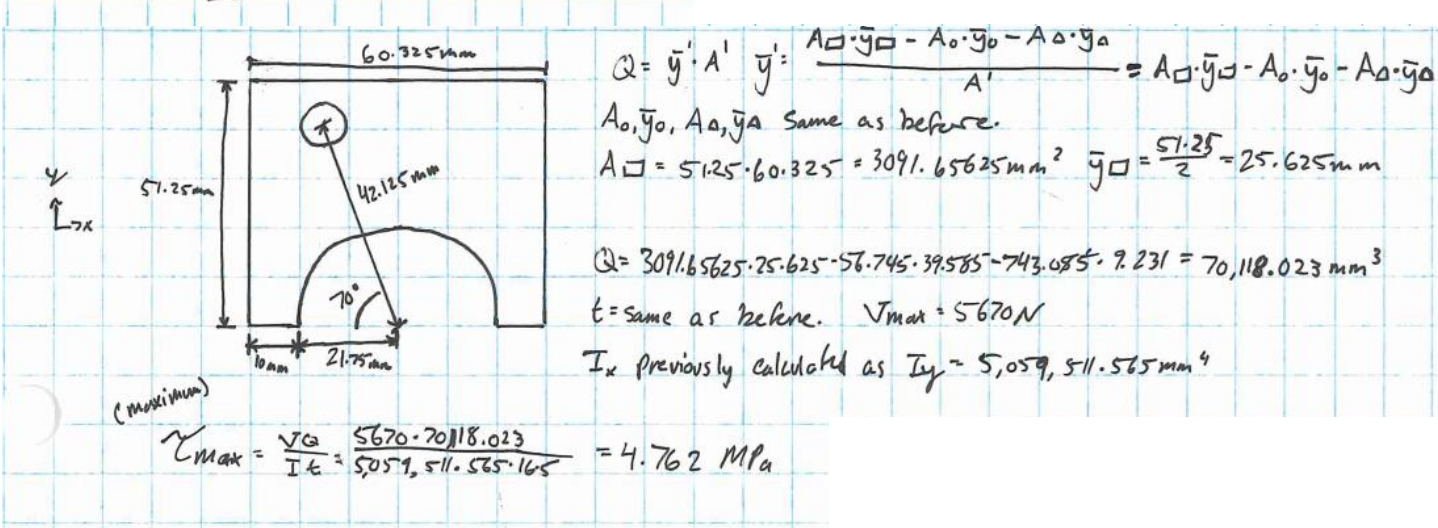
$\sigma_1 = 11.64$ MPa [Tension]

$\sigma_2 = 11.82$ MPa [Compression]

Maximum stresses from towing < minimum stresses from low speed impact.

↳ Chassis won't be damaged by towing at rated weight.





At the centerline the normal stresses due to bending are zero. The maximum direct normal stress has already been determined. For the 3MPH corner impact, $\sigma_{avg} = \frac{N}{A} = \frac{18426.36N}{3200.313mm^2} = 5.758MPa$. Using the equation $\tau_{max\ in-plane} = \sqrt{(\frac{\sigma_x+\sigma_y}{2})^2 + {\tau_{xy}}^2}$ Gives a maximum shear stress of $\sqrt{(\frac{5.758MPa+0MPa}{2})^2 + 4.406MPa^2} = 5.26MPa$. The same analysis is done with the trailer hitch loads resulting as $\sigma_{avg} = \frac{2780N}{4583.6525mm^2} = 0.6065MPa$, and $\tau_{max\ in-plane} = \sqrt{(\frac{0.6065MPa+0MPa}{2})^2 + 4.762MPa^2} = 4.77MPa$. The maximum shear stresses from towing are also less than that of a 3 MPH impact. The shear forces in other directions are minor in comparison and the calculations have been left out of this report.

Future Work

In continuing this project, the primary focus is to further develop the equations governing the forces acting on the tow ball. These new equations should aim to remove as many assumptions and neglected factors as possible, most importantly brake dive, differences in the height of the centre of mass, changes to the tongue weight and variations in tongue to axle distance. There should also be effort put into calculating the forces present under high lateral acceleration and speed bump/pothole analysis. Lastly, a standardized repair patch to fix corrosion on rear bumper mount flanges should be designed as corrosion in this area is quite standard on these aging cars.

Conclusion

This project began with the goal of reducing waste through improving the functionality of an existing vehicle; that goal has been proven possible with calculations to back up the claims of a towing capacity at 1800kg unbraked. This document details the assumptions made, calculations performed, completed design and design features, as well as media to showcase the design and how it would look mounted on the vehicle. The receiver is given an 1800kg rating, although unbraked loads will prove difficult to stop at or near this limit. Unbraked loads are recommended to be limited to 500kg to increase safety, this is an arbitrary decision made to match BMW's towing capacity ratings from [6].

Appendix

Table 1:
Safety standards described by [4].

Standard	Model Year(s) Applicable	Barrier/Pendulum Speed and Parts Affected
FMVSS 215	1973	5 mph front and 2 1/2 mph rear impact with barrier. Safety-related parts only.
FMVSS 215	1974-1978	5 mph front and rear impacts with barrier and pendulum; 3 mph corner impact with pendulum. Safety-related parts only. Pendulum test established bumper height between 16 and 20 inches.
Part 581 incorporating FMVSS 215	1979	As above, plus no damage to exterior surfaces, except bumper facebar and its fasteners.
As above	1980-1982	As above, except face bar can have no permanent deviation in contour or position greater than 3/4 inch, and no permanent localized surface deviation greater than 3/8 inch.
As above	1983 and thereafter	2.5 mph front and rear impacts with barrier and pendulum; 1.5 mph corner impact with pendulum. No damage to safety-related parts and exterior surfaces, except bumper facebar and fasteners.

Table 2:
Acceleration values recorded during testing, measuring device is an iPhone 11 accelerometer [created by author].

Normal Driving		Maximum Values	
Acceleration	0.2g	Acceleration	0.5g
Deceleration	0.3g	Deceleration	0.8g
Left Turn	0.2g	Left Turn	0.6g
Right Turn	0.2g	Right Turn	0.6g

Table 3:
Static coefficient of rolling resistance for basic calculations provided by [1].

	Surface		
Vehicle Type	Concrete	Medium Hard	Sand
Passenger cars	0.015	0.08	0.30
Heavy Trucks	0.012	0.06	0.25
Tractors	0.02	0.04	0.20

Graphs 1 & 2:
Graphs depicting the spring constant vs pressure and load vs stroke [2].

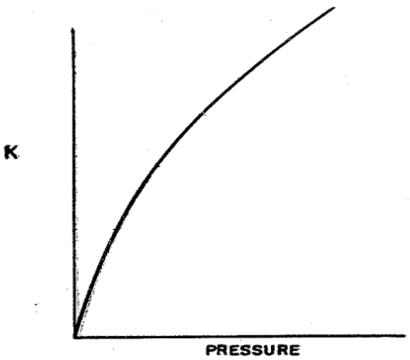


Fig 4

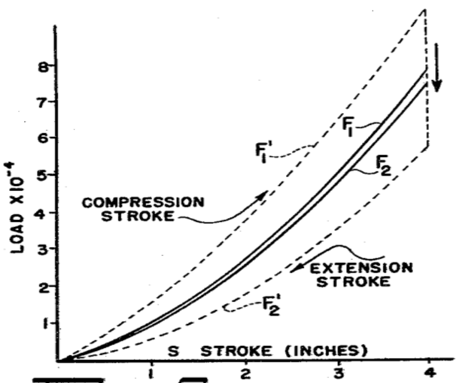
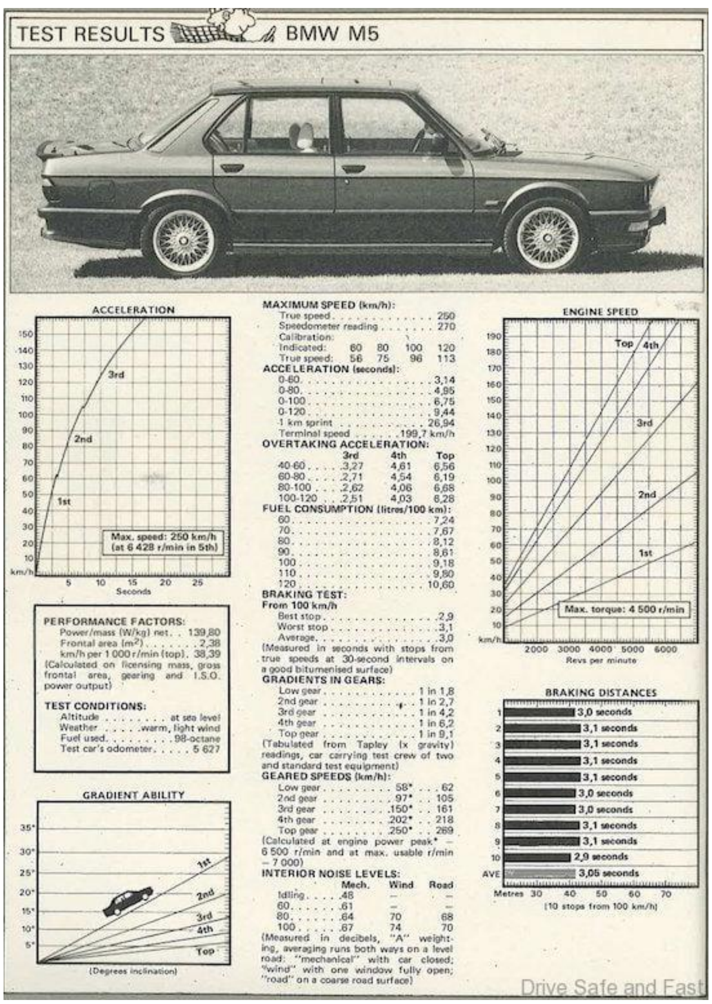


Fig-5

INVENTOR.
GILES A. KENDALL

BY
R.E. Grangue
Attorney

Info sheet 1:
Test data obtained by independent source [3].



TEST RESULTS BMW M5

ACCELERATION

Max. speed: 250 km/h (at 6 428 r/min in 5th)

MAXIMUM SPEED (km/h):
True speed 250
Speedometer reading 270
Calibration:

Indicated:	60	80	100	120
True speed:	56	75	96	113

ACCELERATION (seconds):
0-60 3,14
0-80 4,95
0-100 6,75
0-120 9,44
1 km sprint 26,94
Terminal speed . . . . . . 199,7 km/h

OVERTAKING ACCELERATION:

	3rd	4th	Top
40-60	3,27	4,61	6,56
60-80	2,71	4,54	6,19
80-100	2,62	4,06	6,68
100-120	2,51	4,03	6,28

FUEL CONSUMPTION (litres/100 km):
60 7,24
70 7,67
80 8,12
90 8,61
100 9,18
110 9,80
120 10,60

BRAKING TEST:
From 100 km/h
Best stop 2,9
Worst stop 3,1
Average 3,0
(Measured in seconds with stops from true speeds at 30-second intervals on a good bitumenised surface)

GRADIENTS IN GEARS:
Low gear . . . . 1 in 1,8
2nd gear . . . . 1 in 2,7
3rd gear . . . . 1 in 4,2
4th gear . . . . 1 in 6,2
Top gear . . . . 1 in 9,1
(Tabulated from Tapley (x gravity) readings, car carrying test crew of two and standard test equipment)

GEARED SPEEDS (km/h):
Low gear . . . . 58* . . . 62
2nd gear . . . . 97* . . 105
3rd gear . . . . 150* . . 161
4th gear . . . . 202* . . 218
Top gear . . . . 250* . . 269
(Calculated at engine power peak* – 6 500 r/min and at max. usable r/min – 7 000)

INTERIOR NOISE LEVELS:

	Mech.	Wind	Road
Idling	48	–	–
60	61	–	–
80	64	70	68
100	67	74	70

(Measured in decibels, "A" weighting, averaging runs both ways on a level road: "mechanical" with car closed; "wind" with one window fully open; "road" on a coarse road surface)

PERFORMANCE FACTORS:
Power/mass (W/kg) net . . 139,80
Frontal area (m^2) 2,38
km/h per 1 000 r/min (top) . 38,39
(Calculated on licensing mass, gross frontal area, gearing and I.S.O. power output)

TEST CONDITIONS:
Altitude . . . . at sea level
Weather . . . . . warm, light wind
Fuel used 98-octane
Test car's odometer 5 627

GRADIENT ABILITY
(Degrees inclination)

ENGINE SPEED
Max. torque: 4 500 r/min
Revs per minute

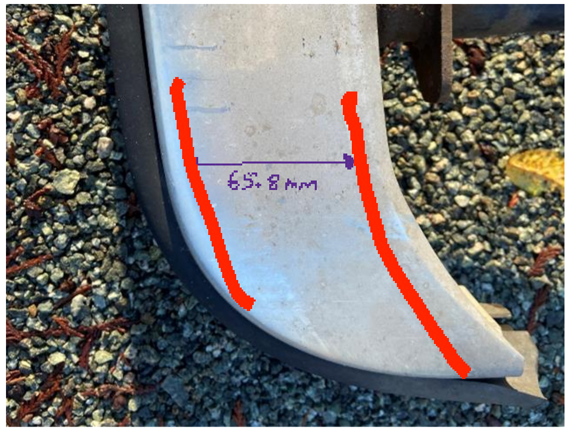
BRAKING DISTANCES

1	3,0 seconds
2	3,1 seconds
3	3,1 seconds
4	3,1 seconds
5	3,1 seconds
6	3,0 seconds
7	3,0 seconds
8	3,1 seconds
9	3,1 seconds
10	2,9 seconds
AVE	3,05 seconds

Metres
(10 stops from 100 km/h)

Photos:

Measurements taken to determine the compression distance of the bumper [created by author].



Examples of measurements taken to determine the shock absorber mounting flange profile [created by author].

Photo of the shock absorber with a patent number [created by author].

Measurement taken to determine curvature radius for lateral acceleration calculations [8].

References

[1] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*. Warrendale, PA: Society of Automotive Engineers, 1993, pp. 59, 60, 117.

[2] G. A. Kendall, *DAMPERS AND DAMPED SPRINGS,* 3053526, Sept 11, 1962. Available: https://image-ppubs.uspto.gov/dirsearch-public/print/downloadPdf/3053526. Accessed: Nov. 30, 2023. [Online].

[3] D. S. Fernandez, "BMW E28 M5 Classic Super Sedan Review", Sept 7 2016. [Online]. Available: https://www.dsf.my/2016/09/bmw-e28-m5-super-sedan-still-the-best/. [Accessed: Nov. 30, 2023].

[4] United States Department of Transportation, National Highway Traffic Safety Administration, *An Evaluation of the Bumper Standard-As Modified in 1982*. Feb 1987. [Online]. Available: https://crashstats.nhtsa.dot.gov/Api/Public/Publication/807072. [Accessed: Nov. 30, 2023].

[5] United States Department of Transportation, National Highway Traffic Safety Administration, *LABORATORY TEST PROCEDURE FOR REGULATION PART 581 Bumper Standard*. Apr 25, 1990. [Online]. Available: https://www.nhtsa.gov/sites/nhtsa.gov/files/tp-581-01.pdf. [Accessed: Nov. 30, 2023].

[6] BMW AG, *525e "Eta"en riiplezier: een stap in de richtin van optimal rendement.* BMW AG, 1983. [Online]. Available: https://autocatalogarchive.com/wp-content/uploads/2023/03/BMW-Serie-5-1983-NL.pdf. [Accessed: Nov. 30, 2023].

[7] rexmo. (2007, Feb 1). 528e automatic towing capacity? [Msg 8]. Message posted to https://www.mye28.com/viewtopic.php?t=28626. [Accessed: Dec. 5, 2023].

[8] Google. "Google Maps". [Online]. Available: https://www.google.com/maps/@48.6754787,-123.4261371,18.91z?entry=ttu. [Accessed: Dec. 5, 2023].